# NAG Toolbox for MATLAB

# f07tu

## 1 Purpose

f07tu estimates the condition number of a complex triangular matrix.

## 2 Syntax

```
[rcond, info] = f07tu(norm_p, uplo, diag, a, 'n', n)
```

## 3 Description

f07tu estimates the condition number of a complex triangular matrix $A$, in either the 1-norm or the $\infty$-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \qquad \text{or} \qquad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{T}}\right)$.

Because the condition number is infinite if $A$ is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham 1988) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

## 4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **norm_p – string**

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

**norm_p** = '1' or 'O'

$\kappa_1(A)$ is estimated.

**norm_p** = 'I'

$\kappa_\infty(A)$ is estimated.

*Constraint*: **norm_p** = '1', 'O' or 'I'.

2: **uplo – string**

Indicates whether $A$ is upper or lower triangular.

**uplo** = 'U'

$A$ is upper triangular.

**uplo** = 'L'

$A$ is lower triangular.

*Constraint*: **uplo** = 'U' or 'L'.

3: **diag – string**

Indicates whether $A$ is a nonunit or unit triangular matrix.

**diag** = 'N'

$A$ is a nonunit triangular matrix.

**diag** = 'U'

$A$ is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

*Constraint*: **diag** = 'N' or 'U'.

4: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The $n$ by $n$ triangular matrix $A$.

If **uplo** = 'U', $A$ is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', $A$ is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of $A$ are assumed to be 1, and are not referenced.

## 5.2 Optional Input Parameters

1: **n – int32 scalar**

*Default*: The second dimension of the array **a**.

$n$, the order of the matrix $A$.

*Constraint*: $\mathbf{n} \geq 0$.

## 5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, rwork

## 5.4 Output Parameters

1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of $A$. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, $A$ is singular to working precision.

2: **info – int32 scalar**

**info** = 0 unless the function detects an error (see Section 6).

# 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** = $-i$

If **info** = $-i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **uplo**, 3: **diag**, 4: **n**, 5: **a**, 6: **lda**, 7: **rcond**, 8: **work**, 9: **rwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value $\rho$, and in practice is nearly always less than $10\rho$, although examples can be constructed where **rcond** is much larger.

## 8 Further Comments

A call to f07tu involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $4n^2$ real floating-point operations but takes considerably longer than a call to f07ts with one right-hand side, because extra care is taken to avoid overflow when $A$ is approximately singular.

The real analogue of this function is f07tg.

## 9 Example

```
norm_p = '1';
uplo = 'L';
diag = 'N';
a = [complex(4.78, +4.56), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(2, -0.3), complex(-4.11, +1.25), complex(0, 0), complex(0,
0);
       complex(2.89, -1.34), complex(2.36, -4.25), complex(4.15, +0.8),
complex(0, 0);
      complex(-1.89, +1.15), complex(0.04, -3.69), complex(-0.02, +0.46),
complex(0.33, -0.26)];
[rcond, info] = f07tu(norm_p, uplo, diag, a)
```

```
rcond =
    0.0268
info =
           0
```